Data Analytics
(CS40003)

Practice Set V

(Topic: Hypothesis Testing)

I. Concept Questions

1. In a hypothesis test, the p value is 0.043. This means that the null hypothesis would be rejected at $\alpha = 0.05$.

2. If the null hypothesis is rejected by a one-tailed hypothesis test, then it will also be rejected by a two-tailed test.

3. If a null hypothesis is rejected at the 0.01 level of significance, it will also be rejected at the 0.05 level of significance.

4. If the test statistic falls in the rejection region, the null hypothesis has been proven to be true.

5. The risk of a type II error is directly controlled in a hypothesis test by establishing a specific significance level.

6. If the null hypothesis is true, increasing only the sample size will increase the probability of rejecting the null hypothesis.

7. If the null hypothesis is false, increasing the level of significance (α) for a specified sample size will increase the probability of rejecting the null hypothesis.

8. A coin is tossed 10,000 times and head turns up 5,195 times. Is the coin unbiased?

9. A sample of 900 members is found to have a mean of 3.47 cm. Can it be reasonably regarded as a simple sample from a large population with mean 3.23 cm. and standard deviation 2.31 cm ?

10. The heights of six randomly chosen sailors are, in inches, 63, 65, 58, 69, 71 and 72. The heights of 10 randomly chosen soldiers are, in inches, 61, 62, 65, 66, 69, 69, 70, 71, 72 and 73. Do these figures indicate that soldiers are on an average shorter than sailors? Test at 5% level of significance.

11. Suppose a test on the hypotheses H0: m = 200 against Ha : m > 200 is done with 1% level of significances p = 40 and n = 16.
What is the probability that the null hypothesis might be accepted when the true mean is really 210? What is the power of the test for m = 210? How these values of b and 1 – b change if the test had used 5% level of significance?
Which is more serious, a Type I and Type II error?

12. The following nine observations were drawn from a normal population:

9 20 24 23 29 21 17 27

(i) Test the null hypothesis H0: m = 26 against the alternative hypothesis Ha: m ¹ 26. At what level of significance can H0 be rejected?
(ii) At what level of significance can H0 : m = 26 be rejected when tested against Ha : m < 26

13. The following sample was taken from a normally distributed population with a known standard deviation $\sigma = 4$. Test the hypothesis that the mean $\mu = 20$ using a level of significance of 0.05 and the alternative that $\mu > 20$:

23, 32, 22, 31, 27, 25, 21, 24, 20, 18.

14. Using the data in Exercise 1 and using a 0.05 level of significance, test the null hypothesis that the population sampled has a mean of $\mu = 171$. Use a two-tailed alternative.

15. Assume that a random sample of size 25 is to be taken from a normal population with $\mu = 10$ and $\sigma = 2$. The value of μ, however, is not known by the person taking the sample.

   a) Suppose that the person taking the sample tests $H0:\mu = 10.4$ against $H1:\mu=10.4$. Although this null hypothesis is not true, it may not be rejected, and a type II error may therefore be committed. Compute β if $\alpha = 0.05$.
   b) Suppose the person wanted to test $H0:\mu = 11.2$ against $H1:\mu = 11.2$.Compute β for $\alpha = 0.05$ and $\alpha = 0.01$.
   c) What principles of hypothesis testing are illustrated by these exercises?

16. A standardized test for a specific college course is constructed so that the distribution of grades should have $\mu = 100$ and $\sigma = 10$. A class of 30 students has a mean grade of 92.

   a. Test the null hypothesis that the grades from this class are a random sample from the stated distribution. (Use $\alpha = 0.05$.)
   b. What is the p value associated with this test?
   c. Discuss the practical uses of the results of this statistical test.

17. The family incomes in a certain city in 1970 had a mean of \$14,200 with a standard deviation of \$2600. A random sample of 75 families taken in 1975 produced $\bar{y}$ = \$15, 300 (adjusted for inflation).

   Assume σ has remained unchanged and test to see whether mean income has changed using a 0.05 level of significance.

18. A drug company is testing a drug intended to increase heart rate. A sample of 100 yielded a mean increase of 1.4 beats per minute, with a standard deviation known to be 3.6. Since the company wants to avoid marketing an ineffective drug, it proposes a 0.001 significance level. Should it market the drug? (Hint: If the drug does not work, the mean increase will be zero.)